EGYPTIAN SARDINIAN TRIGONOMETRY

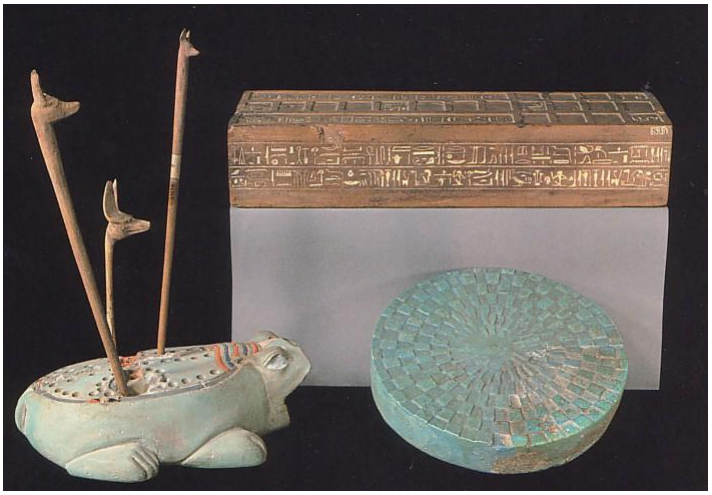


Fig. 1

After a careful observation of the Imenmes Games at the Louvre (fig. 1), it has been possible to reconstruct the highly advanced mathematical and trigonometric achievements in ancient Egypt.
Particularly, on the precious stone turquoise disk there is a deep cutting in the shape of a chessboard that is a true powerful system for polar-exponential representations (in which both angles and radii are exponentially scaled).
This system, probably derived from the inner structure of some citrus fruit, allows a substantial simplification of all mathematical functions.
The same functions, even when less developed, have deep meanings and they can describe the growth of all plants according to their phyllotaxis. Figure 2 shows the growth of basil, mint, and nettle leaves, which are supplementarily coupled, by means of the following simple functions:

$$\mathbf{y1 = x^{1/3},\ y2 = x^{1/2},\ y3 = x^{2/3}}$$

and:

$$\mathbf{y4 = 1/x^{1/3},\ y5 = 1/x^{1/2},\ y6 = 1/x^{2/3}}$$

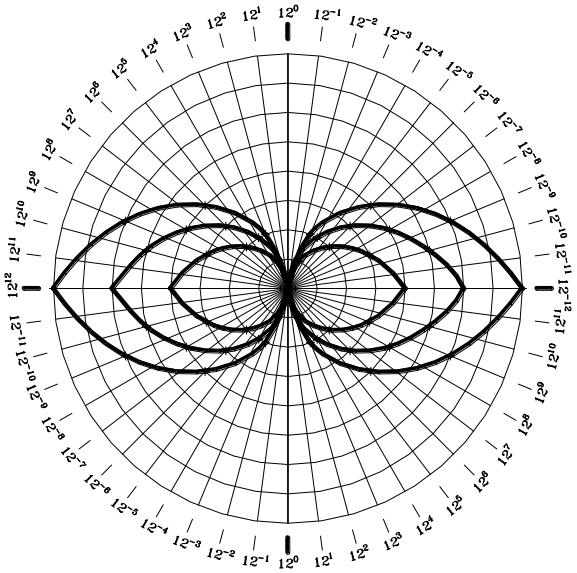


Fig. 2

Within this wonderful system, both the functions **sine** and **cosine** can be represented by four circumferences whose value has never been reconsidered by anyone over the last centuries. The two circumfereces placed along the vertical axis are representative of the function **sine**, while those that are left refer to the function **cosine** (fig. 3).

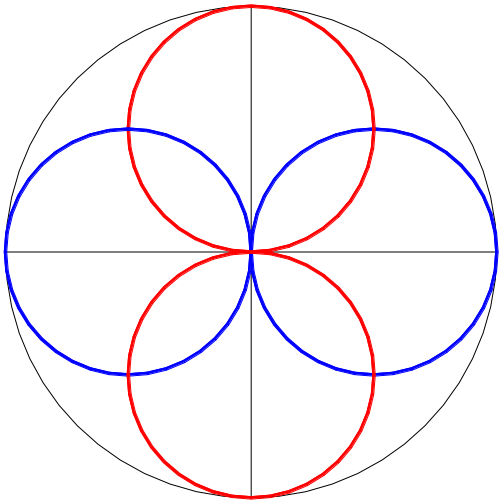


Fig. 3

Figure 4 explains why the horizontal circumferences are representative of the function **cosine**.

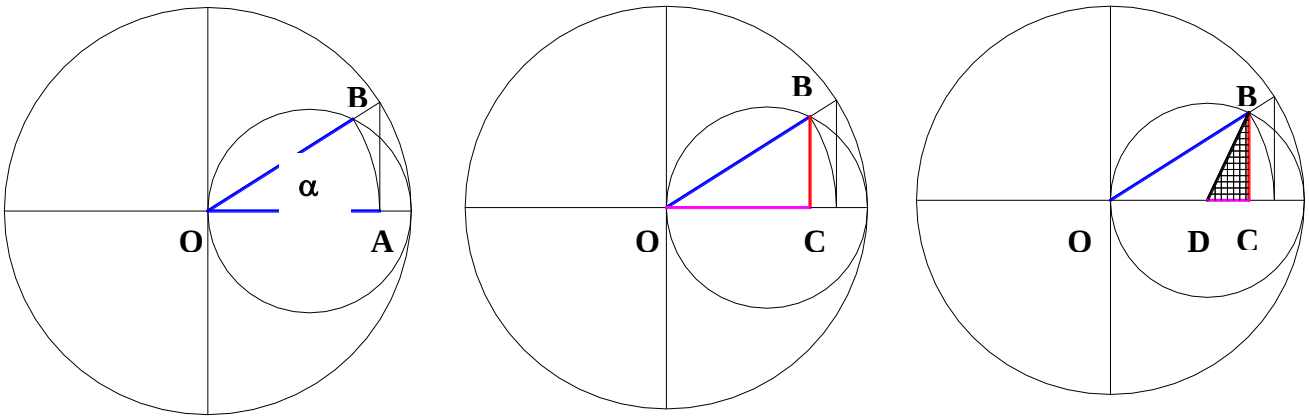


Fig. 4

If we think about exponents, in such a way as to make the scale linear, and if we rotate the segment **OA** up to the starting angle **α**, in a trigonometric circumference with unit radius, we obtain:

$$\mathbf{BC = \sin(\alpha)*\cos(\alpha)}$$

and

$$\mathbf{OC = \cos^2(\alpha)}.$$

But

$$\mathbf{CD = OC - 1/2},$$

then the Pythagorean theorem

$$\mathbf{BC^2 + CD^2 = BD^2}$$

becomes:

$$\mathbf{\sin^2(\alpha)*\cos^2(\alpha) + [\cos^2(\alpha) - 1/2]^2 = 1/4}.$$

Further developments would lead to:

$$\mathbf{\sin^2(\alpha)*\cos^2(\alpha) + \cos^4(\alpha) + 1/4 - \cos^2(\alpha) = 1/4},$$

next:

$$\mathbf{\cos^2(\alpha)*[\ \sin^2(\alpha) + \cos^2(\alpha)] + 1/4 - \cos^2(\alpha) = 1/4}$$

and, since $\sin^2(\alpha) + \cos^2(\alpha) = 1$, we have:

$$\mathbf{\cos^2(\alpha) + 1/4 - \cos^2(\alpha) = 1/4},$$

at last:

$$\mathbf{1/4 = 1/4},$$

independently from α, which proves that the little circumference is a function cosine!
Therefore the solution to all trigonometric problems could be entrusted to a simple game with a string (folded and refolded in two) and two pegs (inserted in the centres of the circumferences). Thanks to this game it is possible to understand the enormous ability of Egyptian rope stretchers described by Herodotus (fig. 5).

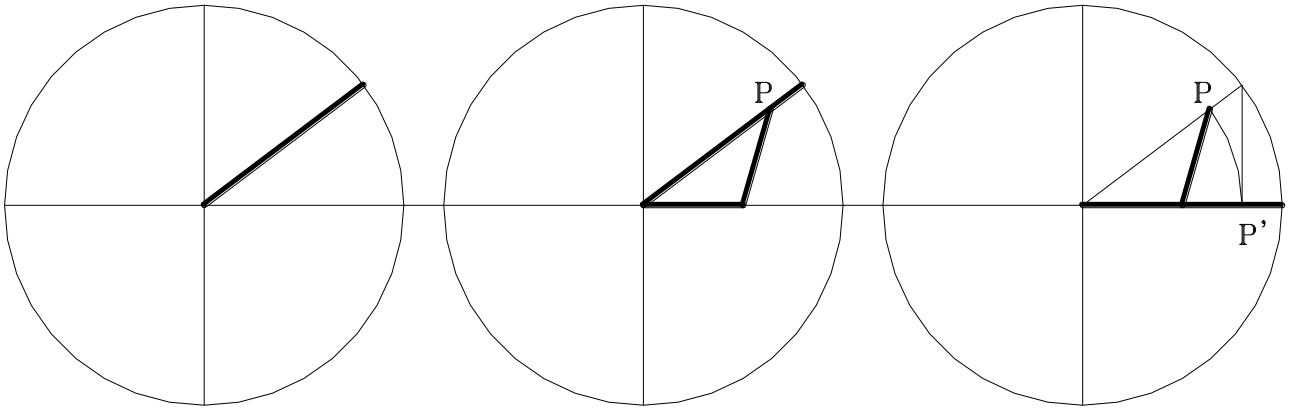


Fig. 5

This representative system has led to the construction of the rotating ruler of Imenmes (fig. 6). This device consists of both a graduated circle, in which four other circumferences are engraved, and a rule overlapping the circle itself.



Fig. 6

If we direct the rule towards any angle, it will be possible to find the value of both functions **sine** and **cosine** in the four circumferences; e.g. for an angle of **30°** (fig. 7) we see on the **sine** circumference the value **0.5** and on the **cosine** circumference a value included between **0.86** and **0.87**.

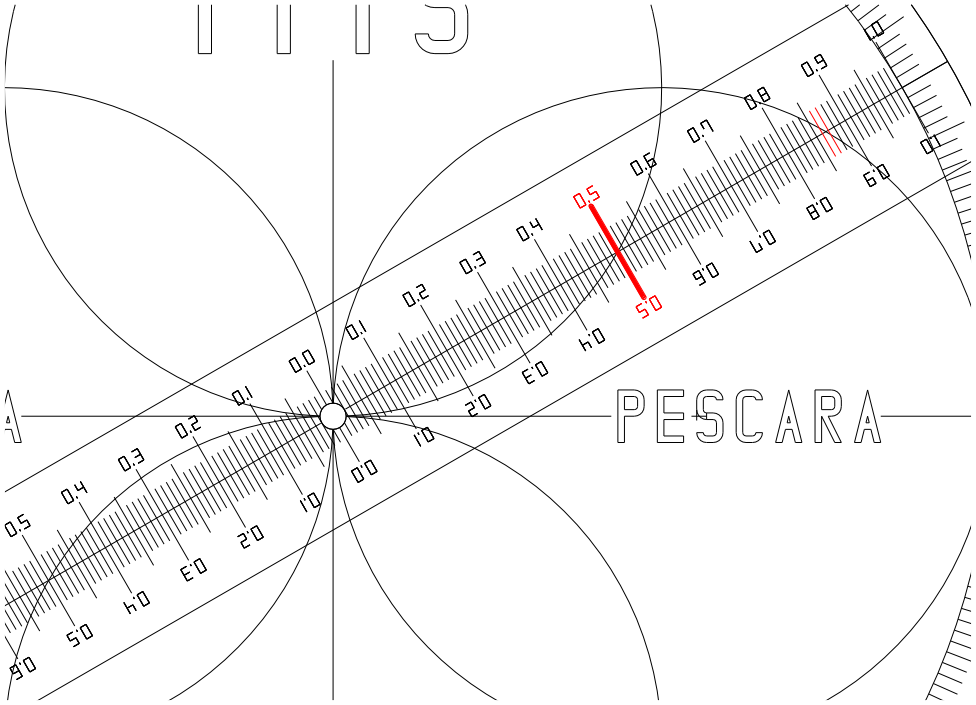


Fig. 7

If we consider an angle of **60°**, the values of both **sine** and **cosine** are obviously inverted (fig. 8).

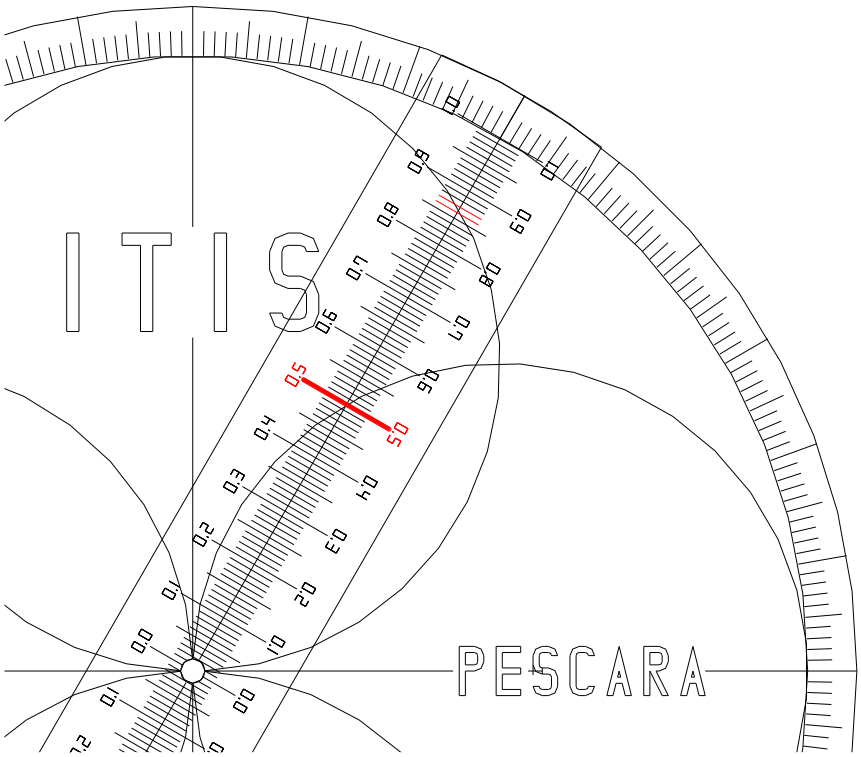


Fig. 8

When observing an angle of **45°** the two funcions have the same value, between **0.70** and **0.71** (fig. 9).

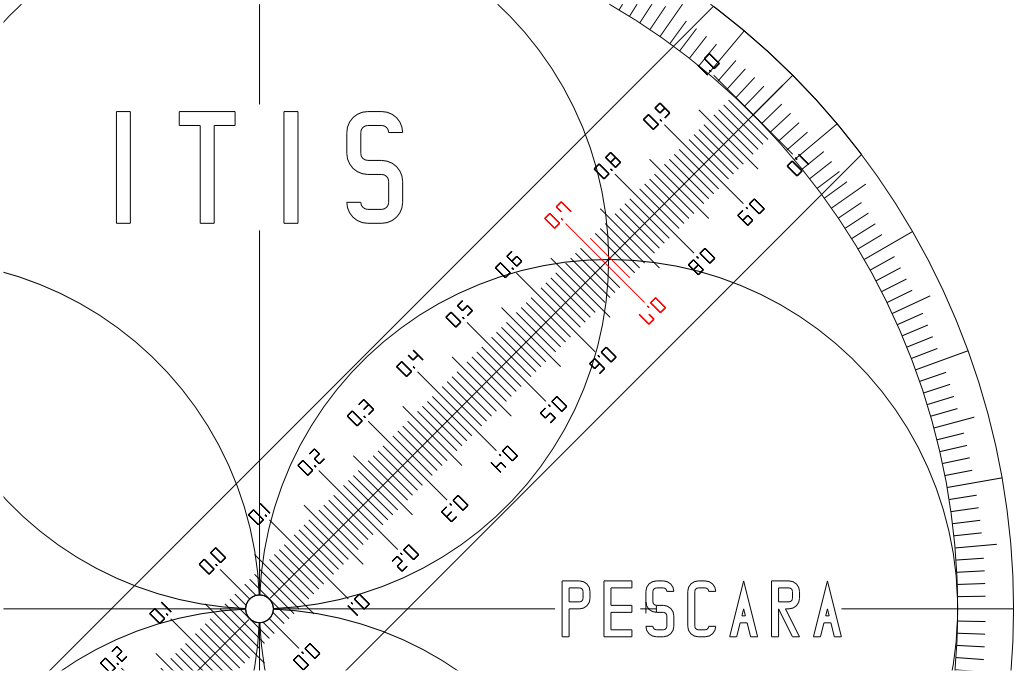


Fig. 9

Calculating the inverse functions **arc sine** and **arc cosine** is an easy and amusing activity: e.g. setting the value **0.3** on the **sine** circumference we univocally point the rule at an angle whose value is between **17°** and **18°**, equal to the **arc sin(0.3)** (fig. 10).

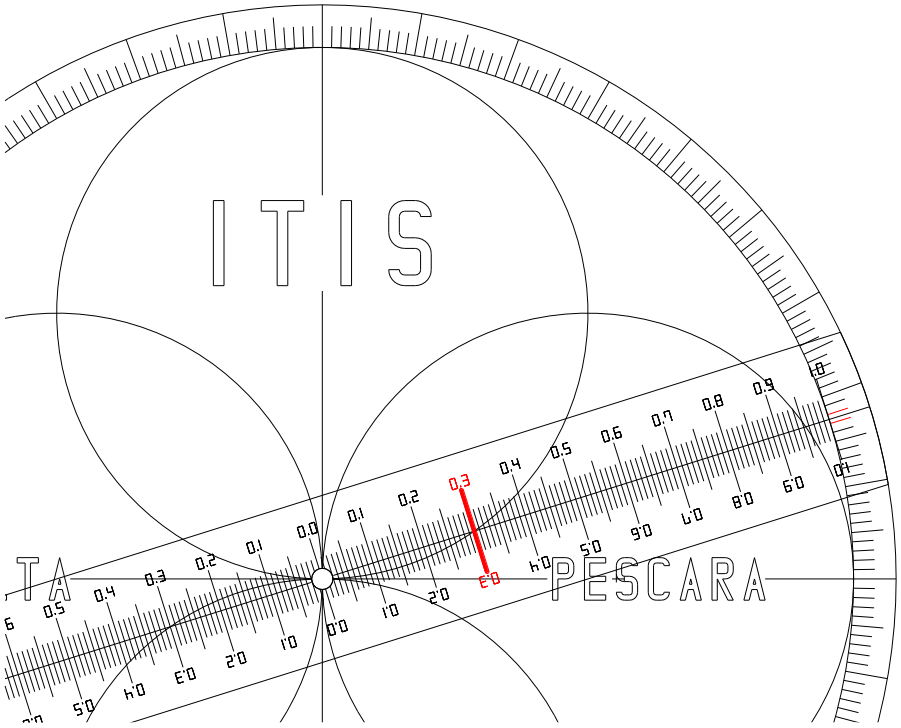


Fig. 10

Even the Pythagorean triples are easily recognizable on the ruler: the triple **3**, **4** and **5** is represented by the numbers **0.6**, **0.8** and **1.0**; in figure 11 they lay on the **sine, cosine** and trigonometric circumferences (but there is an inverted solution).

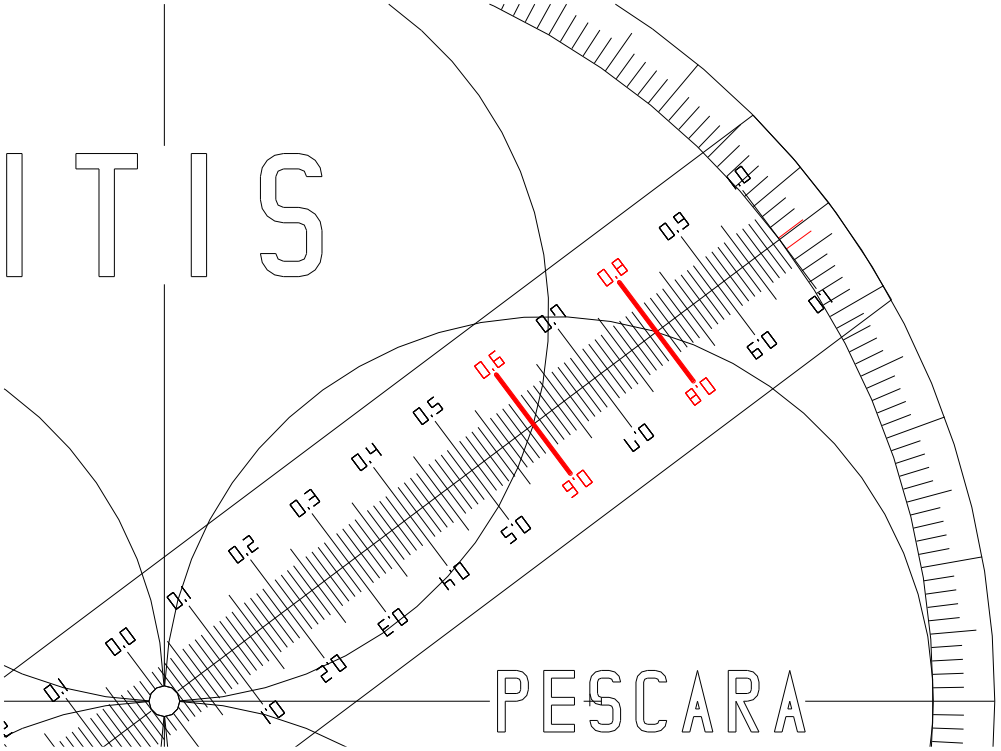


Fig. 11

The same geometrical pattern of the ruler with its typical intersections can be admired on the wonderful painted ceilings of many Theban tombs (fig. 12).



Fig. 12

The Mediterranean roots of Egyptian geometry are proved by various finds belonging to coeval civilizations. This is the case of some Sardinian bronze that portray ancient warriors holding patterned shields (fig. 13). The comparison between the shields and the ruler shows that they both have the same circumferences.



Fig. 13

A bronze statuette, which can be seen at the Museo Pigorini in Rome (fig. 14), clearly shows that Sardinian people knew the golden ratio ϕ. The ruler derived from the circular shield of this bronze allows to move progressively from angles of **36°** to angles of **60°** (**$\cos(36°)/\phi = \cos(60°)$**), and from angles of **60°** to angles of **72°** (**$\cos(60°)/\phi = \cos(72°)$**) which are typical of decagons, hexagons and pentagons.



Fig. 14

In this case the rule obviously has two scales in golden ratio (the packed scale is normally used for the little circumference and the other for the bigger one). If we direct the rule towards an angle of **36°** we find the value of the function **cosine 0.81** (fig. 15).

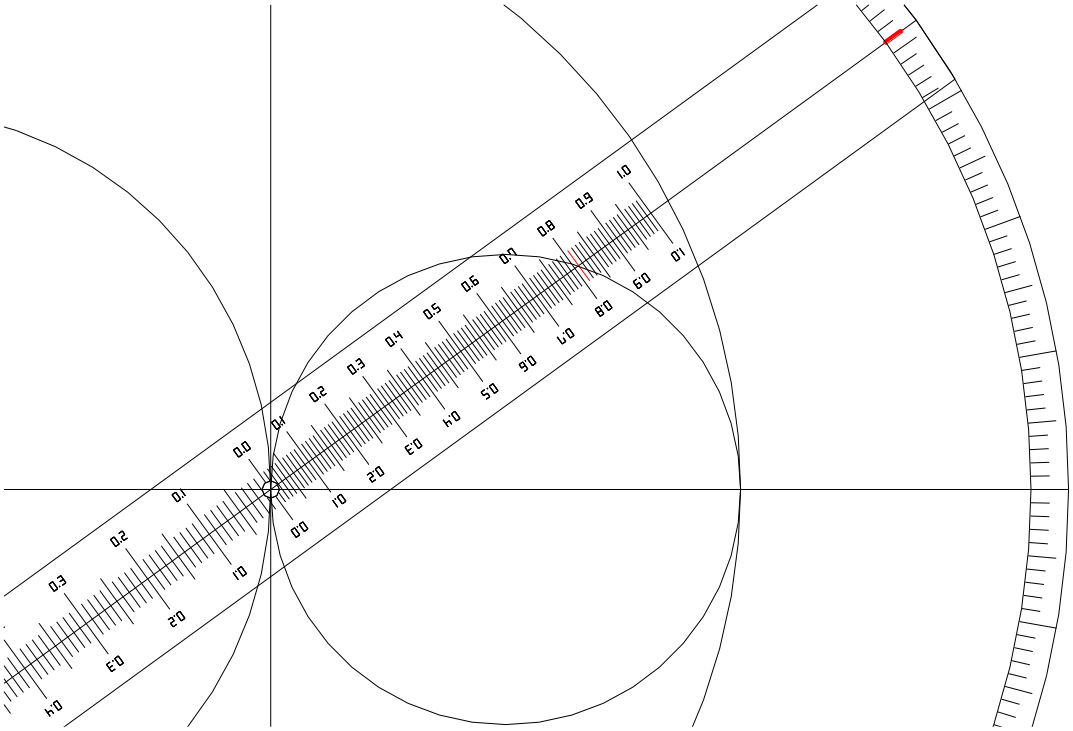


Fig. 15

Changing the scale we reach the value **0.5** (fig. 16).

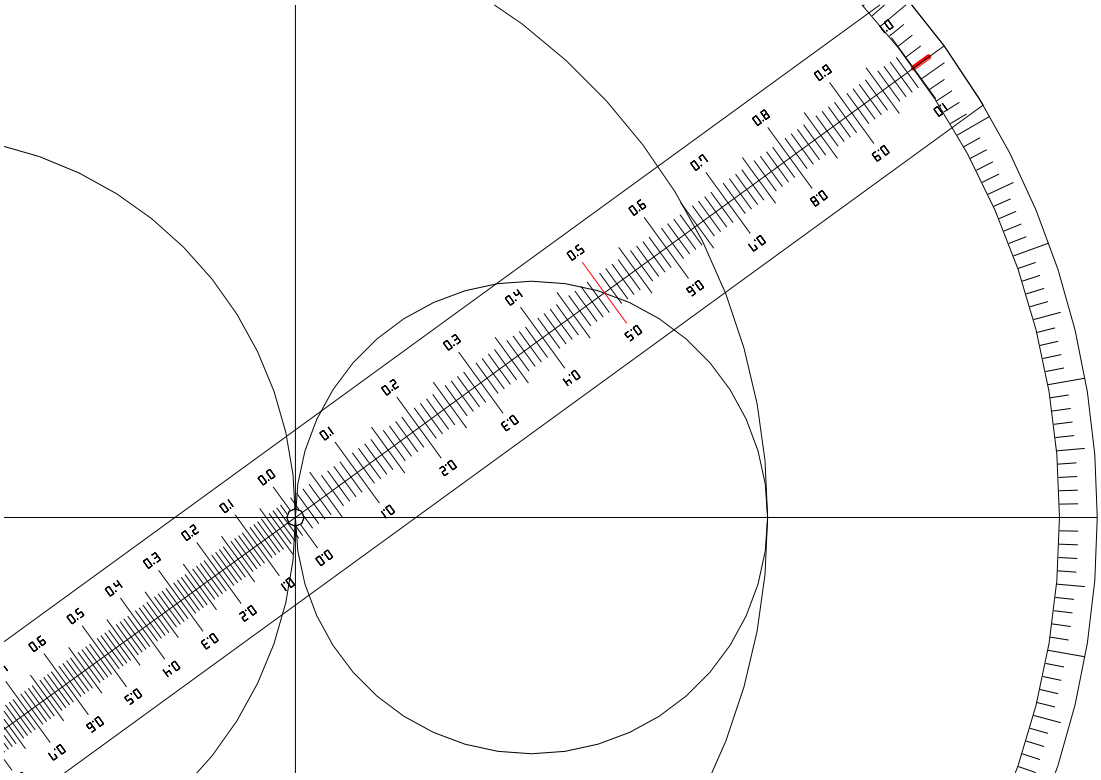


Fig. 16

Setting the value **0.5** on the little circumference (fig. 17) we obtain an angle of **60°** (**arc cos(0.5) = 60°**).

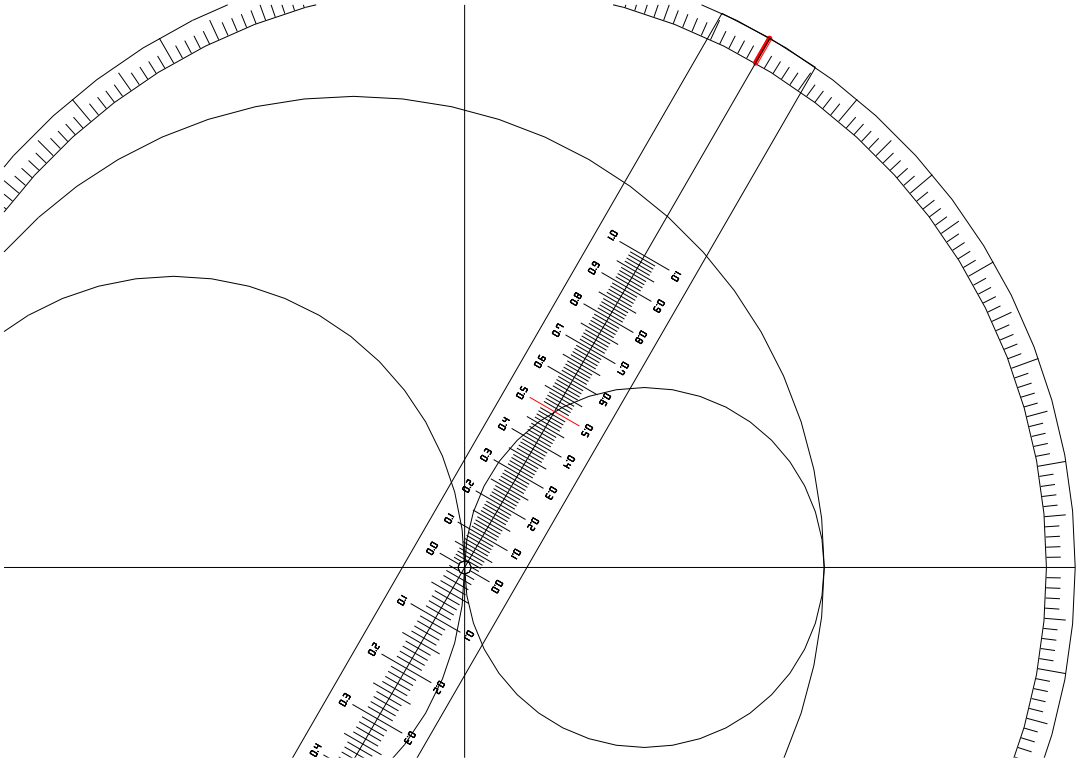


Fig. 17

Changing the scale again we read **0.31** (fig. 18).

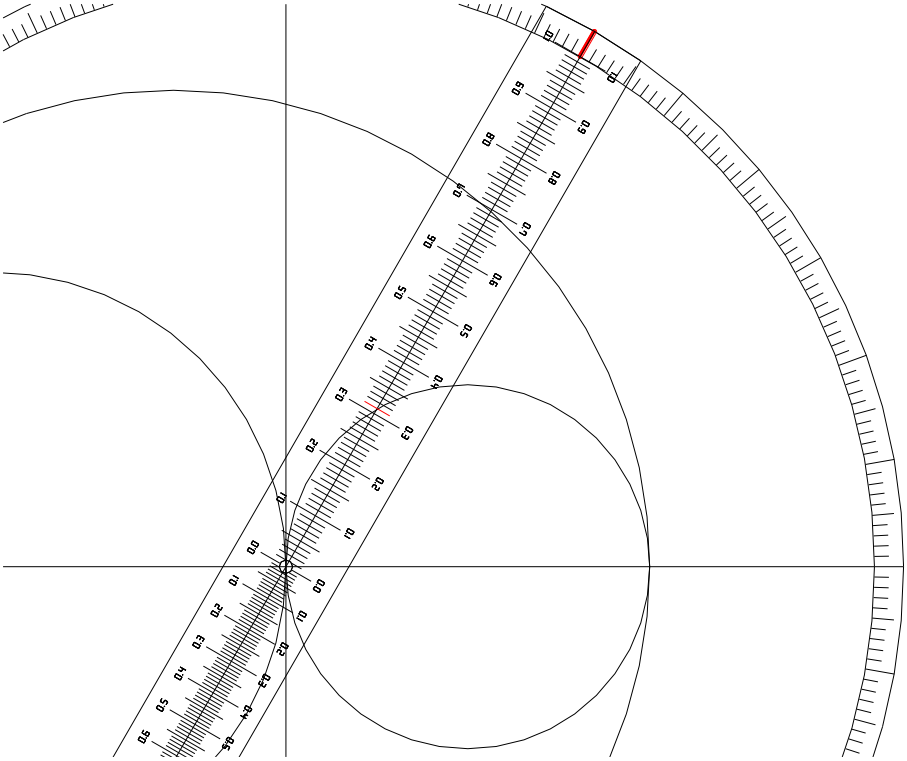


Fig. 18

Setting the last value on the first circumference (fig. 19) we find, at last, the angle **72°** (**arc cos(0.31) = 72°**).

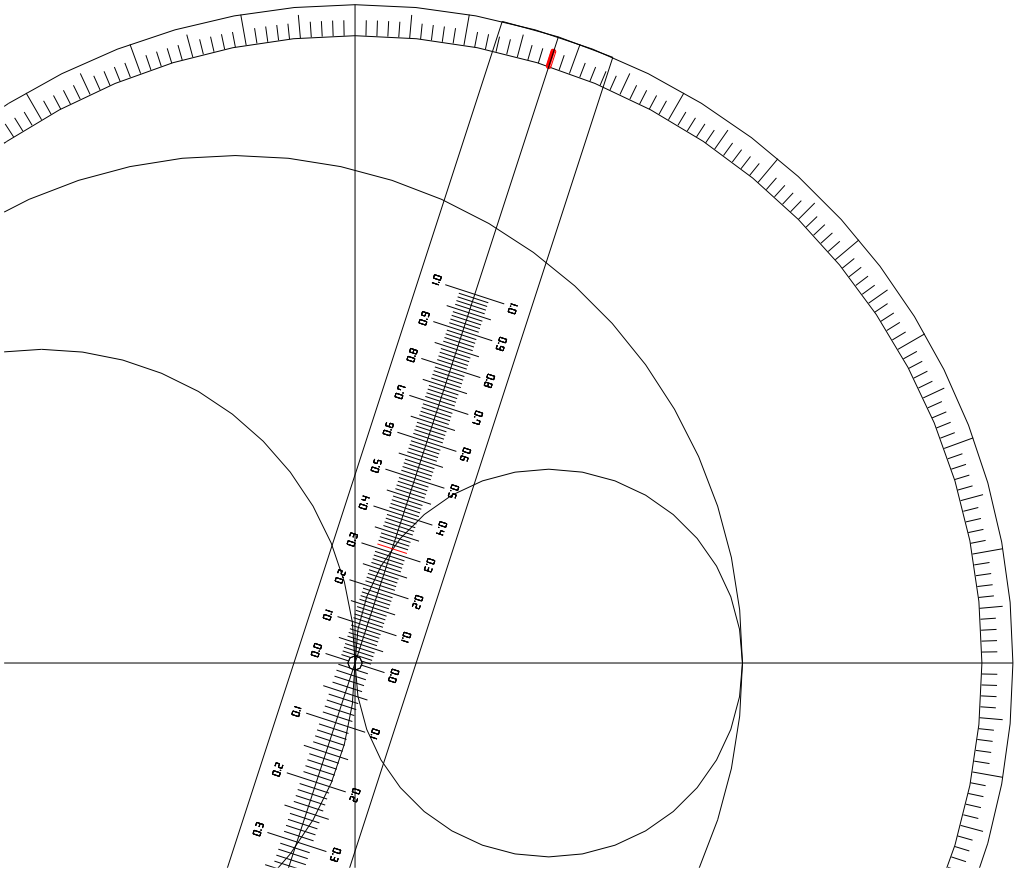


Fig. 19

Another bronze statuette, at the Museo Archeologico in Cagliari, proves the deep trigonometric knowledge of Sardinian people; it shows that an eccentricity makes the **sine** and **cosine** circumferences oval (fig. 20a). Figure 20 b shows the related scientific diagram.



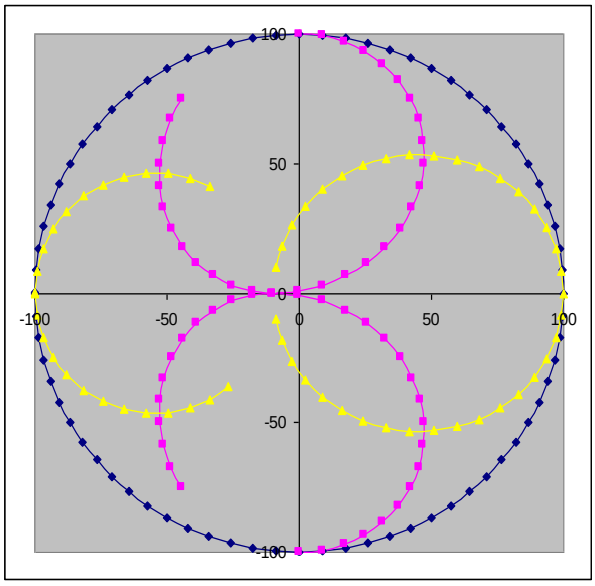


Fig. 20

Thanks to this system, also more complex trigonometric functions could be represented. For example it would be possible to describe the growth of flowers, regardless of the number of their petals. With the help of the function **sin(2α)** and **cos(2α)** it is possible to draw a flower with eight petals (fig. 21).

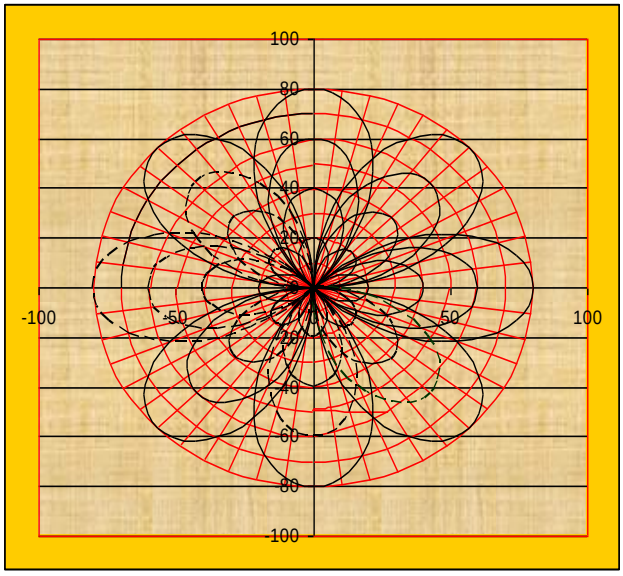


Fig. 21

At the same Museum in Cagliari it is possible to admire two warriors, placed on the same basement, holding two shields with a particular combination of patterns (fig. 22 a). One of the shields is clearly divided in **4** sectors (suggesting the angle **4α**), while the other represents a flower with sixteen petals. Also the functions **sin(4α)** and **cos(4α)** represent a flower with sixteen petals (fig. 22 b). Although we cannot understand how Sardinian people could have used such complex functions, it is undeniable that the above functions are really **sin(4α)** and **cos(4α)**.

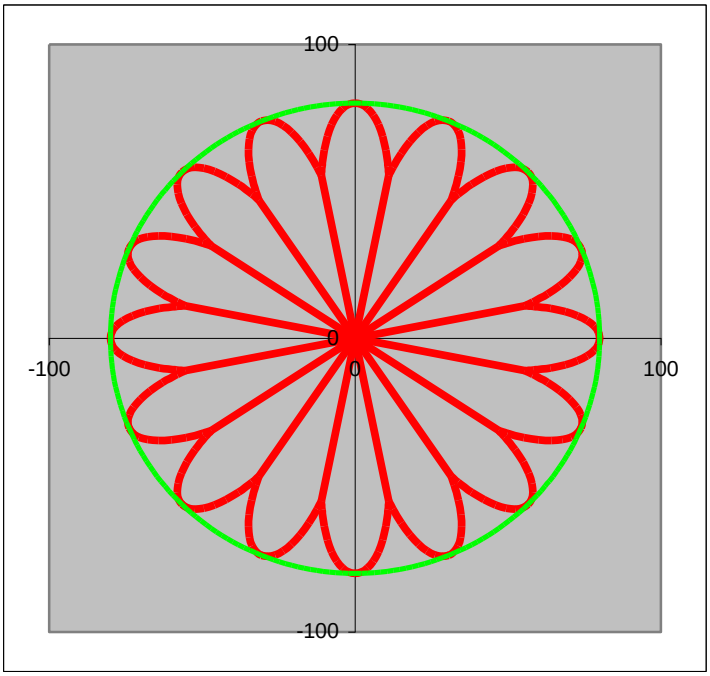


Fig. 22

Some simple combinations of trigonometric functions desribe various organic typologies in details; we obtain the shape of a mallow leaf (fig. 23) properly mixing **cos(0.5α)**, **cos(3.5α)** e **cos(77α)**.

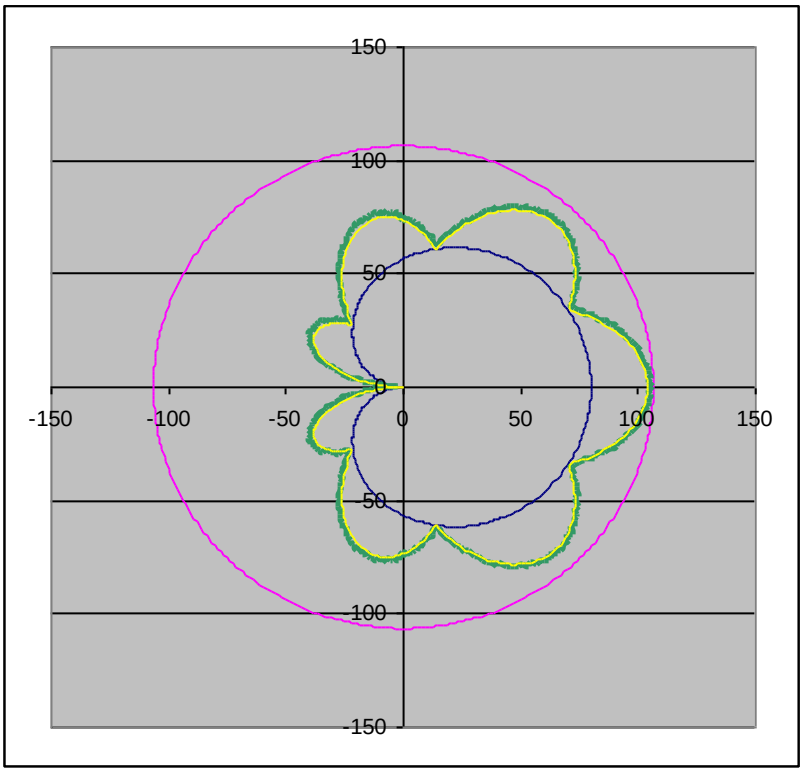


Fig. 23

All the shields held by Sardinian warriors have a right circular cone pointing at a spherical space centre. Why?

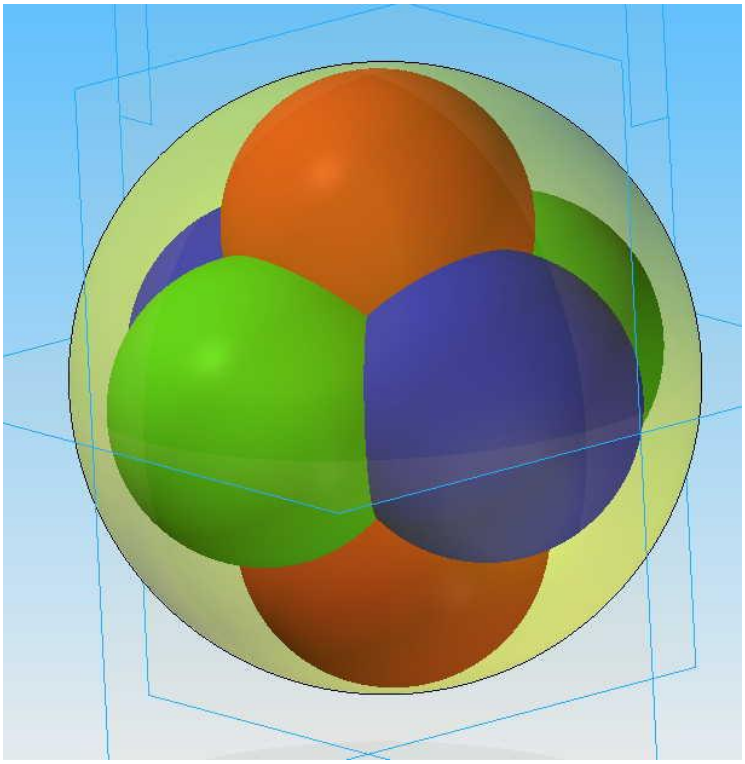


Fig. 24

Four circumferences in the plane become six spheres in the space, which are included in a sphere with unit radius (fig. 24). Any half line coming out from the centre of the trigonometric sphere cuts off the six little spheres thus making the reading of its **direction cosines** immediate.

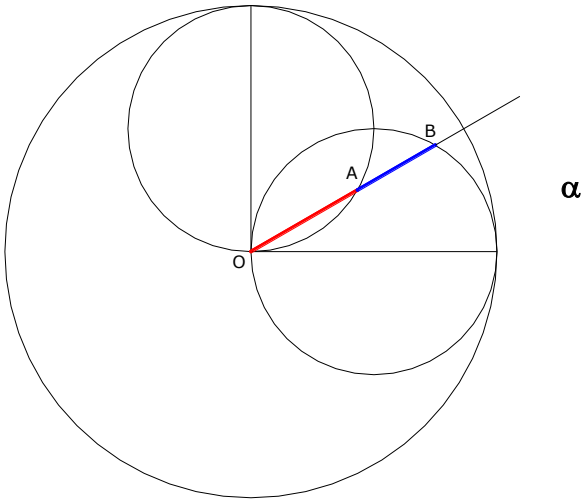


Fig. 25

In the **xy** plane the segment **OA** is **sin(α)** while **OB** represents **cos(α)** (fig. 25). In the **OBz** plan we have (fig. 26 a):

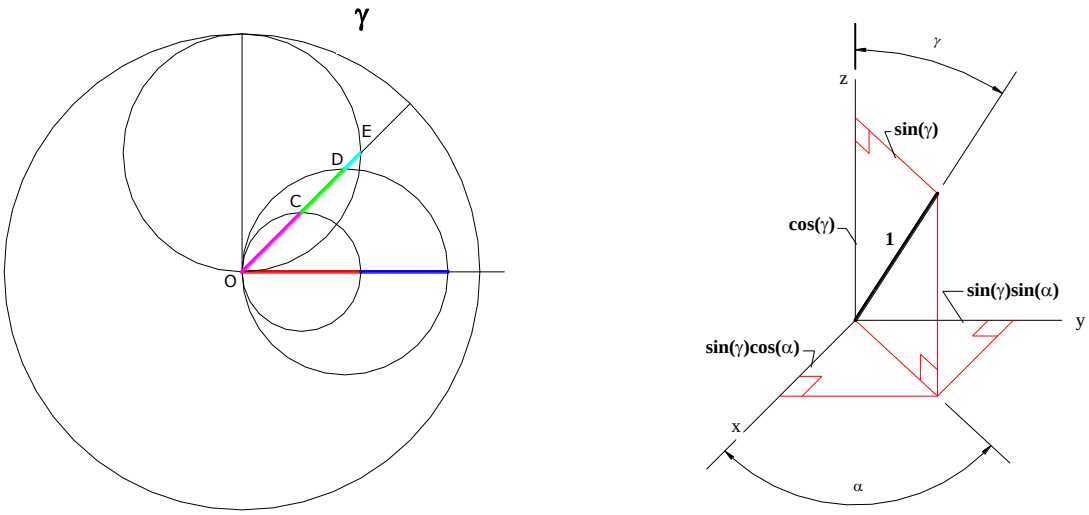


Fig. 26

OC = sin(α)*sin(γ),

OD = cos(α)*sin(γ),

OE = cos(γ).

We can equate (fig. 26 b) in the following way:

$$\mathbf{OC^2 + OD^2 + OE^2 = 1};$$

substituting into this equation the above expressions we obtain:

$$\mathbf{\sin^2(\alpha)*\sin^2(\gamma) + \cos^2(\alpha)*\sin^2(\gamma) + \cos^2(\gamma) = 1},$$

next:

$$\mathbf{\sin^2(\gamma)*[\sin^2(\alpha) + \cos^2(\alpha)] + \cos^2(\gamma) = 1}$$

and since $\sin^2(\alpha) + \cos^2(\alpha) = 1$, this becomes:

$$\mathbf{\sin^2(\gamma) + \cos^2(\gamma) = 1}$$

that is:

$$\mathbf{1 = 1,}$$

which proves that the segments **OC**, **OD** and **OE** are the **direction cosines** and hence these six little spheres are the functions **direction cosines**!

For the above told reasons the Imenmes space can really be considered a true powerful tool of trigonometric representation: it immediately offers the projections, on the three axes **x**, **y** and **z**, of any versor without performig any kind of projection!

The results of this exciting research carried through by Nicolino De Pasquale (at the Technical Institute "A. Volta" in Pescara) were illustrated for the first time on 3rd April 2009, on the occasion of the 50th Anniversary of the above Institute.

The same results were illustrated for the second time at the Science Festival held in Genova in October 2009.

Nicolino De Pasquale

For any didactical purpose please contact prof. Nicolino De Pasquale.